Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year (تخلفات)		Final Term Exam Date: December 26, 2015 Course: Mathematics 1 – A Duration: **3** hours

- The Exam consists of one page
- Answer **All** Questions
- No. of questions: 4
- Total Mark: 100

Question 1

Find **y`** from the following: 24

(a) $y = x^4 + 2^{x^2} + 3x$ (b) $y = \cosh x^2 . \sec 2x$

(c) $y = \cos x^2 - \ln \sin x$ (d) $y = \tan^{-1}x + \tan^{-3}x$

(e) $y^4 = x \log(x + y)$ (f) $y = t \sec t, \ x = t \sinh t$

Question 2

(a)Find the following limits:

(i) $\lim\limits_{x\to 1} \frac{\sqrt{x} - 1}{x^7 - 1}$ (ii) $\lim\limits_{x\to 0} \frac{\ln(1 + 3x)}{2^x - 3^x}$ (iii) $\lim\limits_{x\to 0} \frac{x - \sin x}{x^3 + x^2}$ (iv) $\lim\limits_{x\to \infty} \frac{x^8 + 2x}{x + x^9}$ 12

(b)Write the Maclurin's series of the function: $f(x) = x \sin x$. 4

(c)Show that : $\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$. 4

(d)Determine the extrema of : $f(x) = x^3 - 6x^2$ $g(x) = x^3 + 3$ 6

Question 3

Integrate the following: 30

(a) $\int \frac{x^3}{(4 - x^2)^{1/2}} dx$ (b) $\int \frac{\sin x + \cos x}{\sin x - \cos x} dx$

(c) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$ (d) $\int \tan^4 x \, dx$

(e) $\int x \sqrt{(x^2 + 1)} \, dx$ (f) $\int e^{3x} \sinh 2x \, dx$

Question 4

(a)Find the area of the surface of revaluation generated by revolving about ***x***-axis the cycloid $x = a(t - \sin t), \ y = (1 - \cos t), \ 0 \le t \le 2\pi$ 7

(b) Find the area bounded by the curves: $y = x^3, \ x = 2, \ x = 5, \ y = 0.$ 7

(c) Find the volume generated by revolving, about x-axis, the area bounded by: $y = x^2, \ y = 0, \ x = 10$ 6

Good Luck *Dr. Mohamed Eid* *Dr. Fathi Abdsallam*

Answer

Answer of Question 1

(a)(i) $y` = 4x^3 + 2^{x^2}.\ln 2 . 2x + 3$

(ii) $y` = \cosh x^2 . \sec 2x . \tan 2x . 2 + \sinh x^2 . 2x. \sec 2x$

(iii) $y` = -2x \sin x^2 - \frac{\cos x}{\sin x}$

(iv) $y` = \frac{1}{1+x^2} - 3\ tan^{-4}x. sex^2x$

(v) $4y^3. y` = 1. \log(x+y) + x \frac{1}{\ln 10} . \frac{1+y`}{x+y}$

Then $y` \left(4y^3 - \frac{x}{\ln 10} . \frac{1}{x+y}\right) = \log(x+y) + \frac{x}{\ln 10} . \frac{1}{x+y}$

Then $y` = \frac{\log(x+y) + \frac{x}{\ln 10} . \frac{1}{x+y}}{4y^3 - \frac{x}{\ln 10} . \frac{1}{x+y}}$

(vi) $y` = \frac{\dot{y}}{\dot{x}} = \frac{t \sec t . \tan t + \sec t}{t \cosh t + \sinh t}$

--*24-Marks*

Answer of Question 2

(a)(i) $\lim_{x\to 0} \frac{\sqrt{x} - 1}{x^7 - 1} = \frac{0}{0} = \frac{1/2}{7} = \frac{1}{14}$

(ii) $\lim_{x\to 0} \frac{\ln(1+3x)}{2^x - 3^x} = \frac{0}{0} = \lim_{x\to 0} \frac{\frac{\ln(1+3x)}{x}}{3^x \frac{((2/3)^x - 1)}{x}} = \lim_{x\to 0} \frac{1}{3^x} \frac{\frac{\ln(1+3x)}{x}}{\frac{((2/3)^x - 1)}{x}} = \frac{3}{\ln(2/3)}$

(iii) $\lim_{x\to 0} \frac{x - \sin x}{x^3 + x^2} = \frac{0}{0}$

Using L'Hopital's rule, we get

$\lim_{x\to 0} \frac{x - \sin x}{x^3 + x^2} = \lim_{x\to 0} \frac{1 - \cos x}{3x^2 + 2x} = \frac{0}{0} = \lim_{x\to 0} \frac{\sin x}{6x + 2} = \frac{(0)}{0 + 2} = 0$

(iv) $\lim\limits_{x\to\infty} \dfrac{x^8 + 2x}{x + x^9} = \dfrac{\infty}{\infty} = \lim\limits_{x\to\infty} \dfrac{\dfrac{1}{x} + \dfrac{2}{x^8}}{\dfrac{1}{x^8} + 1} = \dfrac{0+0}{0+1} = 0$

--*12-Marks*

(b)Since $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} ....$

Then $f(x) = x \sin x = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} ....$

--*4-Marks*

(c)Proof

--*4-Marks*

(d) $Since\ \ f`(x) = 3x^2 - 12x = 3x(x-4) = 0$

Then x = 0 , x = 4.

Then, the point (0, 0) is maximum because $f``(0) = 6x - 12 = -12$

the point (4, – 32) is minimum because $f``(4) = 6x - 12 = 12$

$Since\ \ g`(x) = 3x^2 = 0$

Then x = 0 .

Using the first derivative test, we get

$g`(0^-) = g`(-1) = 3x^2 = 3 = g`(1) = g`(0^+)$

Then, the point (0, 3) is neither maximum nor minimum.

--*4-Marks*

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Answer of Question (3)

(a) $\int \dfrac{x^3}{(4-x^2)^{1/2}}dx$ *put* $x = 2\sin\theta \quad \therefore dx = 2\cos\theta d\theta, \quad \cos\theta = \sqrt{4-x^2}$

$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = 2\cos\theta,$

Substitute in the problem we have

$$\therefore \int \frac{x^3}{(4-x^2)^{1/2}}dx = \int \frac{8\sin^3\theta\cos\theta d\theta}{2\cos\theta} = 4\int \sin^3\theta d\theta = 4\int \sin^2\theta.\sin\theta d\theta$$

$$= 4\int\left(1-\cos^2\theta\right)\sin\theta d\theta = 4\int \sin\theta d\theta - 4\int \cos^2\theta\sin\theta d\theta$$

$$= -4\cos\theta + \frac{1}{3}\cos^3\theta + c$$

$$= -4\sqrt{4-x^2} + \frac{1}{3}\left(4-x^2\right)^{3/2} + c$$

(b) $\int \dfrac{\sin x + \cos x}{\sin x - \cos x}dx$

$$\int \frac{\sin x + \cos x}{\sin x - \cos x}dx = \int \frac{\sin x + \cos x}{\sin x - \cos x} \times \frac{\sin x + \cos x}{\sin x + \cos x}dx = \int \frac{(\sin x + \cos x)^2}{\sin^2 x + \cos^2 x}dx$$

$$= \int (\sin x + \cos x)^2 dx = \int (\sin^2 x + 2\sin x\cos x + \cos^2 x dx$$

$$= \int (1 + 2\sin x\cos x)dx = \int (1+\sin 2x)dx = x - \frac{1}{2}\cos 2x + c$$

(c) $\int \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}} dx = \ln(e^{x}+e^{-x})+C$

(d) $\int \tan^{4} x\,dx = \int \tan^{2} x \tan^{2} x dx = \int \tan^{2} x(\sec^{2} x-1)dx$

$$= \int (\tan^{2} x\sec^{2} x - \tan^{2} x)dx$$

$$= \int (\tan^{2} x\sec^{2} x - (\sec^{2} x-1)dx$$

$$= \int (\tan^{2} x\sec^{2} x - \sec^{2} x+1)dx$$

$$= \frac{1}{3}\tan^{3} x - \tan x + x + c$$

$(e) \int x\sqrt{(x^{2}+1)}\,dx$ *put* $y = x^{2}+1$ *then* $dy = 2xdx$

$$\int x\sqrt{(x^{2}+1)}\,dx = \frac{1}{2}\int y^{1/2}dy = \frac{1}{2}.\frac{2}{3}y^{3/2}+C = \frac{1}{3}\left(x^{2}+1\right)^{3/2}+C$$

(f) $\int e^{3x}\cosh 2x\,dx = \int e^{3x}\frac{e^{2x}+e^{-2x}}{2}dx$

$$= \frac{1}{2}\int \left(e^{5x}+e^{x}\right)dx = \left(\frac{1}{10}e^{5x}+\frac{1}{2}e^{x}\right)+c$$

--*30-Marks*

<u>Answer of Question (4)</u>

(a) $x = a(t-\sin t),\ y = a(1-\cos t),\ 0 \le t \le 2\pi$

$x = a(t-\sin t),\ \ y = a(1-\cos t)$ $\frac{dx}{dt} = a(1-\cos t),\ \ \frac{dy}{dt} = a\sin t$

$$dL = \sqrt{\left[\frac{dx}{dt}\right]^{2}+\left[\frac{dy}{dt}\right]^{2}}\,dt = \sqrt{a^{2}(1-\cos t)^{2}+a^{2}\sin^{2} t} = 2a\sin\frac{t}{2}dt$$

$$\therefore S_x = 2\pi\int_a^b y\sqrt{\left[\frac{dx}{dt}\right]^2+\left[\frac{dy}{dt}\right]^2}\,dt = 4\pi a\int_0^{2\pi} a(1-\cos t)\sin\tfrac{t}{2}\,dt$$

$$= 8\pi a^2\int_0^{2\pi}\sin^3\tfrac{t}{2}\,dt = 8\pi a^2\int_0^{2\pi}\sin^2\tfrac{t}{2}\sin\tfrac{t}{2}\,dt$$

$$= 8\pi a^2\int_0^{2\pi}(1-\cos^2\tfrac{t}{2})\sin\tfrac{t}{2}\,dt$$

$$= 8\pi a^2\int_0^{2\pi}(\sin\tfrac{t}{2}-\cos^2\tfrac{t}{2}\sin\tfrac{t}{2})dt$$

$$= -8\pi a^2\left[2\cos\tfrac{t}{2}+\frac{2}{3}\cos^3\tfrac{t}{2}\right]_0^{2\pi} =$$

$$= -8\pi a^2\left[(2\cos\tfrac{\pi}{2}+\tfrac{2}{3}\cos^3\tfrac{\pi}{2})-(2\cos 0+\tfrac{2}{3}\cos^3 0)\right]$$

$$= -8\pi a^2\left[0-(2+\tfrac{2}{3})\right] = \frac{64}{3}a^2\pi$$

(b) $$A = \int_2^5 y\,dx = \int_2^5 x^3 dx = \left[\frac{1}{4}x^4\right]_2^5 = \frac{1}{4}\left[5^4-2^4\right]$$

$$= \frac{625-16}{4} = \frac{609}{4} \text{ square unit}$$

(c) $$V = \pi\int_{x=0}^{x=10} y^2 dx = \pi\int_0^{10} x^4 dx = \frac{\pi}{5}\left[x^5\right]_0^{10} = 20000\pi \text{ cubic unit}$$

--*20-Marks*

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